COURSE DATA

DATA SUBJECT

Code: 44080
Name: Seminar on geometry and topology
Cycle: Master's Degree
ECTS Credits: 3
Academic year: 2025-26

STUDY (S)

Degree	Center	Acad. year	Period
2183 - Master's Degree in Mathematical Research	Facultat de Ciències Matemàtiques	1	Second quarter
2903 - Doble M.U. Prof.Educ.Secund (esp. matem.) e Invest.Matem.	Facultat de Formació del Professorat	1	Second quarter

SUBJECT-MATTER

Degree	Subject-matter	Character
2183 - Master's Degree in Mathematical Research 2903 - Doble M.U. Prof.Educ.Secund (esp. matem.) e Invest.Matem.	Specialty in fundamental mathematics	ELECTIVES

COORDINATION

PEÑAFORT SANCHIS GUILLERMO

SUMMARY

In the first part of this course, we will study affine algebraic sets. These spaces are the basic objects of study in classical algebraic geometry and consist of subsets of n-dimensional affine space defined by polynomial equations. Studying them will allow us to understand the geometric meaning of algebraic notions such as regular elements/zero divisors, Krull dimension, the Noetherian condition, localizations of a ring, and the relationship between prime ideals and irreducible sets. We will conclude this part by proving that the category of affine algebraic sets is equivalent to the category of finitely generated reduced k-algebras, introducing the necessary concepts from category theory.

In the second part, we will study schemes, which are the basic objects in modern algebraic geometry. We will see how the previously mentioned concepts extend to this context and prove that the category of affine schemes is equivalent to the category of rings.

PREVIOUS KNOWLEDGE

RELATIONSHIP TO OTHER SUBJECTS OF THE SAME DEGREE

There are no specified enrollment restrictions with other subjects of the curriculum.

OTHER REQUIREMENTS

Relationship with other courses in the same degree program: many of the algebraic notions studied also appear in the Algebra Seminar.

Other types of requirements: Although basic notions of algebra and topology will be used, the necessary concepts will be introduced, and no prior knowledge is required.

COMPETENCES / LEARNING OUTCOMES

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Que los estudiantes comprendan los conceptos y las demostraciones rigurosas de teoremas fundamentales de alguna de las áreas específicas de las Matemáticas.

Que los estudiantes comprendan los conceptos y las demostraciones rigurosas de teoremas fundamentales de áreas transversales de las Matemáticas.

Que los estudiantes posean la capacidad para enunciar y verificar proposiciones en alguna de las áreas de las Matemáticas y para transmitir los conocimientos matemáticos adquiridos, oralmente y por escrito.

Que los estudiantes sean capaces de aplicar los resultados y técnicas aprendidas para la resolución de problemas complejos de alguna de las áreas de las Matemáticas, en contextos académicos o profesionales.

Que los estudiantes sean capaces de comprender de manera autónoma artículos de investigación o innovación en alguna de las áreas de las Matemáticas.

Que los estudiantes tengan capacidad para elaborar y desarrollar razonamientos lógico-matemáticos e identificar errores en razonamientos incorrectos.

Students should apply acquired knowledge to solve problems in unfamiliar contexts within their field of study, including multidisciplinary scenarios.

Students should demonstrate self-directed learning skills for continued academic growth.

DESCRIPTION OF CONTENTS

1. Affine algebraic sets: Hilbert¿s Nullstellensatz. Zariski topology. Dimension and irreducible components. Regular functions, germs, morphisms, the duality between algebraic sets and finitely generated reduced k-

algebras.

2. Commutative algebra: Rings, domains, fields, R-algebras. Maximal, prime, and radical ideals. Krull dimension, graded rings, and localization of rings.

3. Category theory: Objects and morphisms, functors and natural transformations. Hom functors and equivalences of categories.

4. Schemes: The spectrum of a ring. Zariski topology and dimension. The structure sheaf, germs. Morphisms and the duality between the category of affine schemes and the category of rings.

WORKLOAD

PRESENCIAL ACTIVITIES

Activity	Hours
Theory	30,00
Total hours	**30,00**

NON PRESENCIAL ACTIVITIES

Activity	Hours
Attendance at other activities	0,00
Individual or group project	20,00
Independent study and work	15,00
Preparation of lessons	0,00
Preparation for assessment activities	10,00
Resolution of case studies	0,00
Total hours	**45,00**

TEACHING METHODOLOGY

Lectures and problem-solving sessions. The course notes include a schedule divided into sessions for independent study.

EVALUATION

Continuous assessment based on the completion of exercises.

REFERENCES

- R. Hartshorne. Algebraic Geometry. Springer-Verlag, 1977.

- M. F. Atiyah, I. G. MacDonald. Introduction to Commutative Algebra. Westview Press, 1994.

- E. Riehl. Category theory in context. Courier Dover Publications, 2017.