COURSE DATA

DATA SUBJECT

Code: 44087
Name: Seminar on applied mathematics
Cycle: Master's Degree
ECTS Credits: 3
Academic year: 2025-26

STUDY (S)

Degree	Center	Acad. year	Period
2183 - Master's Degree in Mathematical Research	Facultat de Ciències Matemàtiques	1	Second quarter
2903 - Doble M.U. Prof.Educ.Secund (esp. matem.) e Invest.Matem.	Facultat de Formació del Professorat	1	Second quarter

SUBJECT-MATTER

Degree	Subject-matter	Character
2183 - Master's Degree in Mathematical Research 2903 - Doble M.U. Prof.Educ.Secund (esp. matem.) e Invest.Matem.	Specialty in applied mathematics	ELECTIVES

COORDINATION

YAÑEZ AVENDAÑO DIONISIO FELIX

SUMMARY

Hyperbolic systems of conservation laws consist of first-order partial differential equations with a special structure that allows for (weak) discontinuous solutions to associated Cauchy problems. These systems appear in many scientific models to express the conservation of relevant quantities in these models.

These equations can be solved analytically in very few cases, thus numerical methods are necessary for approximating these solutions. The challenge faced by these numerical methods is approximating solutions with discontinuities using classical techniques that assume smoothness of the solution.

This course studies the Riemann problem for a scalar conservation law with concave or convex flux. It examines the structure of its weak solutions: the Rankine-Hugoniot condition, shocks, rarefaction waves, and entropic solutions. It also explores linear systems and some two-equation systems, such as the Saint-Venant equations for shallow water flow.

In a second part, finite difference schemes for approximating solutions of conservation law systems are studied, along with basic tools for their analysis (local error, von Neumann stability, Courant-Friedrichs-Lewy condition). The Lax-Wendroff theorem is proved, which states that conservative methods provide adequate approximation. Finite volume methods based on exact Riemann solvers are introduced, with the Godunov method as a prototype, as well as methods based on approximate Riemann solvers like the Roe method.

In the third part, the advantages of higher-order methods such as the Lax-Wendroff method are discussed, and it is shown that linear schemes of order higher than one develop some form of spurious oscillations. This motivates the introduction of schemes with flux limiters or slope limiters.

PREVIOUS KNOWLEDGE

RELATIONSHIP TO OTHER SUBJECTS OF THE SAME DEGREE

There are no specified enrollment restrictions with other subjects of the curriculum.

OTHER REQUIREMENTS

Basic knowledge of finite difference methods for differential equations.

COMPETENCES / LEARNING OUTCOMES

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Que los estudiantes comprendan los conceptos y las demostraciones rigurosas de teoremas fundamentales de alguna de las áreas específicas de las Matemáticas.

Que los estudiantes sean capaces de comprender de manera autónoma artículos de investigación o innovación en alguna de las áreas de las Matemáticas.

Que los estudiantes sean capaces de construir, interpretar, analizar y validar modelos matemáticos avanzados que simulen situaciones reales.

Que los estudiantes sean capaces de diseñar, desarrollar e implementar programas informáticos eficientes para abordar problemas relacionados con las Matemáticas y sus aplicaciones.

Que los estudiantes sean capaces de seleccionar un conjunto de técnicas numéricas, lenguajes y herramientas matemáticas adecuadas para resolver un modelo matemático que simule un problema real.

Que los estudiantes sean capaces de validar e interpretar los resultados obtenidos, comparando con visualizaciones, medidas experimentales y/o requisitos funcionales del correspondiente sistema físico.

Que los estudiantes sepan elegir y utilizar herramientas informáticas adecuadas para abordar problemas relacionados con las Matemáticas y sus aplicaciones.

Que los estudiantes tengan capacidad para elaborar y desarrollar razonamientos lógico-matemáticos e identificar errores en razonamientos incorrectos.

DESCRIPTION OF CONTENTS

1. Hyperbolic systems of conservation laws.

- Riemann problem for a scalar conservation law with concave or convex flux.
- Weak solutions: the Rankine-Hugoniot condition, shocks, rarefaction waves, entropic solutions.
- Linear systems.
- Two-equation systems: Saint-Venant equations for shallow water flow.

2. Numerical methods for scalar conservation laws.

- Finite difference schemes: Lax-Friedrichs method, local error, von Neumann stability, Courant-Friedrichs-Lewy condition.
- Conservative methods: Lax-Wendroff theorem.
- Finite volume methods.
- Exact Riemann solvers: Godunov method.
- Approximate Riemann solvers: Roe method.

3. High-order methods for scalar conservation laws.

- Lax-Wendroff method
- Godunov's theorem
- High-order methods based on flux limiters
- High-order methods based on slope limiters

WORKLOAD

PRESENCIAL ACTIVITIES

Activity	Hours
Theory	30,00
Total hours	**30,00**

NON PRESENCIAL ACTIVITIES

Activity	Hours
Attendance at other activities	0,00
Individual or group project	15,00
Independent study and work	20,00
Preparation of lessons	10,00
Preparation for assessment activities	0,00
Resolution of case studies	0,00

	Total hours	45,00

TEACHING METHODOLOGY

Combination of master class, student presentations on selected topics, and practical sessions in computer labs.

EVALUATION

The evaluation is based on the presentation of selected topics and computer lab exercises.

REFERENCES

- R. LeVeque, 'Finite Difference Methods for Ordinary and Partial Differential Equations, Steady State and Time Dependent Problems'. SIAM 2007

- R. LeVeque 'Numerical Methods for Conservation Laws'. Lectures in Mathematics, ETG-Zurich (1990)

- J. Strikwerda, 'Finite Difference Schems and Partial Differential Equations', Wadsworth &Brooks/Cole (1989)

- E. F. Toro, 'Riemann Solvers and Numerical Methods for Fluid Dynamics', 3rd edition, Springer, 2009.

- E. Godlewski, P.A. Raviart, 'Numerical Approximation of Hyperbolic Systems of Conservation Laws', Springer, 1996.